STA 100A SSI 2023 – Final

August 2, 2023

Name: ____________________

Student ID: ____________________

- Testing time: 90 minutes (from 12:20 to 1:50 pm).
- Format: Closed-book, calculator allowed, 5 written answer questions, each with multiple parts.
- Scoring: Each question is worth 20 points.
- Scratch work: One page provided for scratch work.
- Required information: Print your **name** and fill in your student **ID** number (nine digits).
- Space limitation: Written answers and supporting work must **fit in the provided spaces**.
- Supporting work: Answers without supporting work may not receive credit.
- Submission: **Remove scratch paper, formula sheet, and statistical tables** before submission.

1. Body mass index (BMI) is a measure of how fit a person is, and is given in units of kg/m^2. BMI for all people in a certain state in the US is believed to be normally distributed, with mean 25.44, and standard deviation 4.88. Use this information to complete the following problems.

   (a) Find the probability that a randomly selected subject has a BMI over 23.

   (b) If a randomly selected subject has a BMI under 25.44, what is the probability that their BMI is over 21.78?

   (c) Suppose subjects in the top 25% of the distribution are considered to be "overweight". What is the cutoff for a subject being considered "overweight"?

(d) If we randomly sample six subjects, what is the probability that exactly four of them are "overweight"?

(e) If we randomly sample 25 subjects, what is the probability that at least 10 of them are "overweight"? Apply a normal approximation with continuity correction to solve this problem.

2. The number of hours of sleep per night was measured for subjects who smoked and did not smoke, with the following summary statistics:

	Smokers	Non-smokers
Sample mean	7	6
Sample standard deviation	1.2	1.6
Sample size	60	40

You may assume a random sample was taken from each group, and that the appropriate degrees of freedom are $\nu = 100$. Assume the question of interest is if the average hours of sleep differ between smokers (μ_1) and non-smokers (μ_2).

(a) Find the 99% confidence interval for $\mu_1 - \mu_2$.

(b) Interpret your interval in terms of the problem, being as specific as you can.

(c) What does your confidence interval suggest about the range of the p-value for the hypothesis test $H_0 : \mu_1 - \mu_2 = 0$ v.s. $H_A : \mu_1 - \mu_2 \neq 0$? **Using only information about your confidence interval, explain your answer.**

(d) What conclusions can you draw about the probability of your confidence interval in (a) covering $\mu_1 - \mu_2$? Please explain your answer.

3. According to an official statement released by the state of California, the average height of a Redwood tree in California is less than 240 ft. To assess this claim, a scientist collects a sample of 61 randomly chosen Redwood trees in California. She finds that the average heights in her sample is 235 ft, with a standard deviation 36 ft. Conduct a hypothesis test to assess whether the true average height is less than 240 ft. Use $\alpha = 0.05$.

   (a) State the null and alternative hypotheses.

   (b) Calculate the test statistic.

(c) Find the range of p-value.

(d) Do you reject or fail to reject the null?

(e) What type of error could you have made in this test? Please explain your answer.

4. A statistics professor believes the proportion of males taking his class is 60%, and the proportion of females taking his class is 40%. For a class of size 500, he finds that 270 are male, and 230 are female.

   (a) State the null and alternative hypotheses for the professor's belief.

   (b) Calculate the test statistic.

(c) Find the range of p-value.

(d) State your conclusion in terms of the problem, using $\alpha = 0.01$.

(e) Were there more or less females taking the class than the professor expected? Please explain your answer.

5. A particular type of cherry tree was given three different methods of care (I, II, and III). The fruit yield in pounds was measured for one year, with the following summary statistics.

Care	I	II	III
$\bar{Y}_i$	10	13	13
s_i	5	4	4
n_i	31	31	31

Test whether there is a significant difference among the three different methods of care in terms of the mean fruit yield. Denote the population mean fruit yield with Care I, II, and III as μ_1, μ_2, and μ_3, separately. Assume the population standard deviations for the fruit yield are equal for different methods of care.

(a) Calculate MSB and MSW.

(b) State the null and alternative hypotheses.

(c) Calculate the test statistic.

(d) After performing the test in (b) using a computer, we obtained a p-value of 0.009596. Now, someone claims that the probability of the null hypothesis being true is 0.009596. Do you agree with this statement? Please explain your answer.

(e) What problem would arise if we opted for repeated t tests instead of ANOVA? How can we address this issue without using ANOVA? Please explain your answer.

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Formula Sheet

- Sample mean and variance:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

- Median:

$$(0.5)(n+1)$$

- Addition rule:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

- Multiplication rule:

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2|E_1)$$

- Conditional probability:

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

- Rules of total probability:

$$P(E_1) = P(E_2) \times P(E_1|E_2) + P(E_2^C) \times P(E_1|E_2^C)$$

- Expected value:

$$\mu_Y = \sum y_i P(Y = y_i)$$

- Variance:

$$\sigma_Y^2 = \sum (y_i - \mu_Y)^2 P(Y = y_i)$$

- Linear combinations of random variables:

$$\mu_{aX+b} = a\mu_X + b, \quad \sigma^2_{aX+b} = a^2\sigma^2_X$$

- Binomial $B(n, p)$:

  *
  $$P(Y = j) = \binom{n}{j} p^j (1-p)^{n-j},$$

  where

  $$\binom{n}{j} = \frac{n!}{j!(n-j)!} \text{ and } n! = n(n-1)\cdots 1$$

  *
  $$\mu_Y = np, \quad \sigma_Y^2 = np(1-p)$$

- Standardization:

$$Z = \frac{1}{\sigma_Y}(Y - \mu_Y)$$

- Sampling distribution of the sample mean:

$$\mu_{\bar{Y}} = \mu, \sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}}$$

- Normal approximation: $Y \sim B(n,p)$ can be approximated by $N(np, np(1-p))$
- Continuity correction: $P(Y = n) = P(n - 0.5 < Y < n + 0.5)$
- $1-\alpha$ confidence interval for μ:
  - ∗ Two-sided: $\bar{Y} \pm t_{n-1}(\alpha/2) \times \text{SE}_{\bar{Y}}$
  - ∗ Upper one-sided: $(-\infty, \bar{Y} + t_{n-1}(\alpha) \times \text{SE}_{\bar{Y}})$
  - ∗ Lower one-sided: $(\bar{Y} - t_{n-1}(\alpha) \times \text{SE}_{\bar{Y}}, \infty)$

  where

$$\text{SE}_{\bar{Y}} = \frac{s}{\sqrt{n}}$$

- $1-\alpha$ confidence interval for $\mu_1 - \mu_2$:
  - ∗ Two-sided: $(\bar{Y}_1 - \bar{Y}_2) \pm t_\nu(\alpha/2) \times \text{SE}_{\bar{Y}_1-\bar{Y}_2}$
  - ∗ Upper one-sided: $(-\infty, (\bar{Y}_1 - \bar{Y}_2) + t_\nu(\alpha) \times \text{SE}_{\bar{Y}_1-\bar{Y}_2})$
  - ∗ Lower one-sided: $((\bar{Y}_1 - \bar{Y}_2) - t_\nu(\alpha) \times \text{SE}_{\bar{Y}_1-\bar{Y}_2}, \infty)$

  where the degrees of freedom ν will be given and

$$\text{SE}_{\bar{Y}_1-\bar{Y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- One-sample t test:

H_0	H_A	Test statistic	Rejection region	p-value
$\mu = c$	$\mu \neq c$	$T = \frac{\bar{Y}-c}{\text{SE}_{\bar{Y}}} \overset{H_0}{\sim} t_{n-1}$	$\lvert T\rvert > t_{n-1}(\alpha/2)$	$2 \times P(t_{n-1} > \lvert T\rvert)$
$\mu \geq c$	$\mu < c$		$T < -t_{n-1}(\alpha)$	$P(t_{n-1} < T)$
$\mu \leq c$	$\mu > c$		$T > t_{n-1}(\alpha)$	$P(t_{n-1} > T)$

- Two-sample t test:

H_0	H_A	Test statistic	Rejection region	p-value
$\mu_1 - \mu_2 = c$	$\mu_1 - \mu_2 \neq c$	$T = \frac{(\bar{Y}_1-\bar{Y}_2)-c}{\text{SE}_{\bar{Y}_1-\bar{Y}_2}} \overset{H_0}{\sim} t_\nu$ with	$\lvert T\rvert > t_\nu(\alpha/2)$	$2 \times P(t_\nu > \lvert T\rvert)$
$\mu_1 - \mu_2 \geq c$	$\mu_1 - \mu_2 < c$		$T < -t_\nu(\alpha)$	$P(t_\nu < T)$
$\mu_1 - \mu_2 \leq c$	$\mu_1 - \mu_2 > c$	$\nu = \frac{(\text{SE}_1^2+\text{SE}_2^2)^2}{\text{SE}_1^4/(n_1-1)+\text{SE}_2^4/(n_2-1)}$	$T > t_\nu(\alpha)$	$P(t_\nu > T)$

- Comparison of paired samples: calculate the difference between each pair of observations and then perform inference (confidence interval and t test) on these differences as if it were a one-sample analysis.
- 95% confidence interval for p:
  - ∗ Two-sided: $\tilde{p} \pm 1.96 \times \text{SE}_{\tilde{p}}$
  - ∗ Upper one-sided: $(0, \tilde{p} + 1.645 \times \text{SE}_{\tilde{p}})$
  - ∗ Lower one-sided: $(\tilde{p} - 1.645 \times \text{SE}_{\tilde{p}}, 1)$

  where

$$\tilde{p} = \frac{Y+2}{n+4}, \quad \text{SE}_{\tilde{p}} = \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$$

- 95% confidence interval for $p_1 - p_2$:

$$(\tilde{p}_1 - \tilde{p}_2) \pm 1.96 \times \text{SE}_{\tilde{p}_1 - \tilde{p}_2}$$

where

$$\tilde{p}_1 = \frac{Y_1 + 1}{n_1 + 2}, \quad \tilde{p}_2 = \frac{Y_2 + 1}{n_2 + 2}, \quad \text{SE}_{\tilde{p}_1 - \tilde{p}_2} = \sqrt{\frac{\tilde{p}_1(1 - \tilde{p}_1)}{n_1 + 2} + \frac{\tilde{p}_2(1 - \tilde{p}_2)}{n_2 + 2}}$$

- Chi-square goodness-of-fit test:

$$T = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i} \overset{H_0}{\sim} \chi^2_{k-1},$$

H_0 is rejected at the α level of significance if

$$p\text{-value } = P(\chi^2_{k-1} > T) < \alpha \text{ or } T > \chi^2_{k-1}(\alpha)$$

- Chi-square test of independence:

$$T = \sum_{i=1}^{r \times k} \frac{(o_i - e_i)^2}{e_i} \overset{H_0}{\sim} \chi^2_{(r-1)(k-1)},$$

H_0 is rejected at the α level of significance if

$$p\text{-value } = P(\chi^2_{(r-1)(k-1)} > T) < \alpha \text{ or } T > \chi^2_{(r-1)(k-1)}(\alpha)$$

- ANOVA:

Source	**df**	**SS** (Sum of Squares)	**MS** (Mean Square)
Between groups	$I - 1$	$\sum_{i=1}^{I} n_i(\bar{Y}_i - \bar{Y})^2$	SS/df
Within groups	$n - I$	$\sum_{i=1}^{I} (n_i - 1)s_i^2$	SS/df
Total	$n - 1$	$\sum_{i=1}^{I} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y})^2$	

where

$$\bar{Y} = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} Y_{ij}}{n} = \frac{\sum_{i=1}^{I} n_i \bar{Y}_i}{n}, \quad n = \sum_{i=1}^{I} n_i$$

- F test:

$$T = \frac{\text{MSB}}{\text{MSW}} \overset{H_0}{\sim} F_{I-1, n-I},$$

H_0 is rejected at the α level of significance if

$$p\text{-value } = P(F_{I-1,n-I} > T) < \alpha \text{ or } T > F_{I-1,n-I}(\alpha)$$

- The Bonferonni-adjusted $1 - \alpha$ confidence interval for $\mu_a - \mu_b$ is

$$(\bar{Y}_a - \bar{Y}_b) \pm t_{n-I}(\alpha/(2k)) \times \text{SE}_{\bar{Y}_a - \bar{Y}_b},$$

where

$$\text{SE}_{\bar{Y}_a - \bar{Y}_b} = \sqrt{\text{MSW} \times \left(\frac{1}{n_a} + \frac{1}{n_b} \right)}$$

- Correlation coefficient:

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}}{s_X} \right) \left(\frac{Y_i - \bar{Y}}{s_Y} \right)$$

- Fitted regression line:

$$\hat{Y} = b_0 + b_1 X$$

  where

$$b_1 = r\frac{s_Y}{s_X}, \quad b_0 = \bar{Y} - b_1\bar{X}$$

  - Residuals: $e_i = Y_i - \hat{Y}_i$ where $\hat{Y}_i = b_0 + b_1 X_i$
  - Error sum of suares: $\text{SSE} = \sum_{i=1}^{n} e_i^2$
  - Residual standard deviation: $s_e = \sqrt{\frac{\text{SSE}}{n-2}}$

- Coefficient of determination:

$$r^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2 - \sum_{i=1}^{n}(Y_i - \hat{Y}_i)}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} = 1 - \frac{\text{SSE}}{(n-1)s_Y^2}$$

- $1 - \alpha$ confidence interval for β_1:

$$b_1 \pm t_{n-2}(\alpha/2) \times \text{SE}_{b_1},$$

  where

$$\text{SE}_{b_1} = \frac{s_e}{s_X\sqrt{n-1}}$$

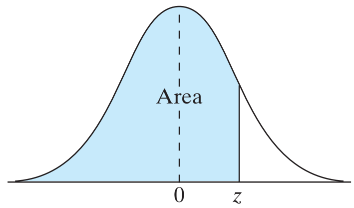
- Test of $H_0 : \beta_1 = 0$ or $H_0 : \rho = 0$:

$$T = \frac{b_1 - 0}{\text{SE}_{b_1}} = r\sqrt{\frac{n-2}{1-r^2}}$$

  H_0 is rejected at the α level of significance if

$$p\text{-value } = 2 \times P(t_{n-2} > |T|) < \alpha \text{ or } |T| > t_{n-2}(\alpha/2)$$

Table 3 Areas Under the Normal Curve

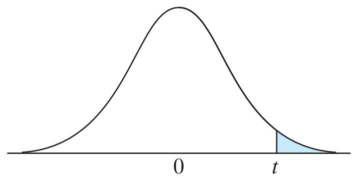


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table 3 Areas Under the Normal Curve *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table 4 Critical Values of Student's *t* distribution



	UPPER TAIL PROBABILITY									
df	0.20	0.10	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924
4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587
11	0.876	1.363	1.796	1.928	2.096	2.201	2.328	2.718	3.106	4.437
12	0.873	1.356	1.782	1.912	2.076	2.179	2.303	2.681	3.055	4.318
13	0.870	1.350	1.771	1.899	2.060	2.160	2.282	2.650	3.012	4.221
14	0.868	1.345	1.761	1.888	2.046	2.145	2.264	2.624	2.977	4.140
15	0.866	1.341	1.753	1.878	2.034	2.131	2.249	2.602	2.947	4.073
16	0.865	1.337	1.746	1.869	2.024	2.120	2.235	2.583	2.921	4.015
17	0.863	1.333	1.740	1.862	2.015	2.110	2.224	2.567	2.898	3.965
18	0.862	1.330	1.734	1.855	2.007	2.101	2.214	2.552	2.878	3.922
19	0.861	1.328	1.729	1.850	2.000	2.093	2.205	2.539	2.861	3.883
20	0.860	1.325	1.725	1.844	1.994	2.086	2.197	2.528	2.845	3.850
21	0.859	1.323	1.721	1.840	1.988	2.080	2.189	2.518	2.831	3.819
22	0.858	1.321	1.717	1.835	1.983	2.074	2.183	2.508	2.819	3.792
23	0.858	1.319	1.714	1.832	1.978	2.069	2.177	2.500	2.807	3.768
24	0.857	1.318	1.711	1.828	1.974	2.064	2.172	2.492	2.797	3.745
25	0.856	1.316	1.708	1.825	1.970	2.060	2.167	2.485	2.787	3.725
26	0.856	1.315	1.706	1.822	1.967	2.056	2.162	2.479	2.779	3.707
27	0.855	1.314	1.703	1.819	1.963	2.052	2.158	2.473	2.771	3.690
28	0.855	1.313	1.701	1.817	1.960	2.048	2.154	2.467	2.763	3.674
29	0.854	1.311	1.699	1.814	1.957	2.045	2.150	2.462	2.756	3.659
30	0.854	1.310	1.697	1.812	1.955	2.042	2.147	2.457	2.750	3.646
40	0.851	1.303	1.684	1.796	1.936	2.021	2.123	2.423	2.704	3.551
50	0.849	1.299	1.676	1.787	1.924	2.009	2.109	2.403	2.678	3.496
60	0.848	1.296	1.671	1.781	1.917	2.000	2.099	2.390	2.660	3.460
70	0.847	1.294	1.667	1.776	1.912	1.994	2.093	2.381	2.648	3.435
80	0.846	1.292	1.664	1.773	1.908	1.990	2.088	2.374	2.639	3.416
100	0.845	1.290	1.660	1.769	1.902	1.984	2.081	2.364	2.626	3.390
140	0.844	1.288	1.656	1.763	1.896	1.977	2.073	2.353	2.611	3.361
1000	0.842	1.282	1.646	1.752	1.883	1.962	2.056	2.330	2.581	3.300
∞	0.842	1.282	1.645	1.751	1.881	1.960	2.054	2.326	2.576	3.291
	60%	80%	90%	92%	94%	95%	96%	98%	99%	99.9%
	CRITICAL VALUE FOR CONFIDENCE LEVEL									

Table 9 Critical Values of the Chi-Square Distribution

Note: Column headings are non-directional (omni-directional) *P*-values. If H_A is directional (which is only possible when df = 1), the directional *P*-values are found by dividing the column headings in half.

	TAIL PROBABILITY						
df	0.20	0.10	0.05	0.02	0.01	0.001	0.0001
1	1.64	2.71	3.84	5.41	6.63	10.83	15.14
2	3.22	4.61	5.99	7.82	9.21	13.82	18.42
3	4.64	6.25	7.81	9.84	11.34	16.27	21.11
4	5.99	7.78	9.49	11.67	13.28	18.47	23.51
5	7.29	9.24	11.07	13.39	15.09	20.51	25.74
6	8.56	10.64	12.59	15.03	16.81	22.46	27.86
7	9.80	12.02	14.07	16.62	18.48	24.32	29.88
8	11.03	13.36	15.51	18.17	20.09	26.12	31.83
9	12.24	14.68	16.92	19.68	21.67	27.88	33.72
10	13.44	15.99	18.31	21.16	23.21	29.59	35.56
11	14.63	17.28	19.68	22.62	24.72	31.26	37.37
12	15.81	18.55	21.03	24.05	26.22	32.91	39.13
13	16.98	19.81	22.36	25.47	27.69	34.53	40.87
14	18.15	21.06	23.68	26.87	29.14	36.12	42.58
15	19.31	22.31	25.00	28.26	30.58	37.70	44.26
16	20.47	23.54	26.30	29.63	32.00	39.25	45.92
17	21.61	24.77	27.59	31.00	33.41	40.79	47.57
18	22.76	25.99	28.87	32.35	34.81	42.31	49.19
19	23.90	27.20	30.14	33.69	36.19	43.82	50.80
20	25.04	28.41	31.41	35.02	37.57	45.31	52.39
21	26.17	29.62	32.67	36.34	38.93	46.80	53.96
22	27.30	30.81	33.92	37.66	40.29	48.27	55.52
23	28.43	32.01	35.17	38.97	41.64	49.73	57.08
24	29.55	33.20	36.42	40.27	42.98	51.18	58.61
25	30.68	34.38	37.65	41.57	44.31	52.62	60.14
26	31.79	35.56	38.89	42.86	45.64	54.05	61.66
27	32.91	36.74	40.11	44.14	46.96	55.48	63.16
28	34.03	37.92	41.34	45.42	48.28	56.89	64.66
29	35.14	39.09	42.56	46.69	49.59	58.30	66.15
30	36.25	40.26	43.77	47.96	50.89	59.70	67.63