# STA 100 Homework 2 Solution

Yidong Zhou

1. The following uses the standard normal table (Z table).
   (a) $P(Z < 1.2) = 0.8849$.
   (b) $P(Z \geq -2.3) = 1 - P(< -2.3) = 1 - 0.0107 = 0.9893$.
   (c) $P(-1 < Z \leq 0.31) = P(Z \leq 0.31) - P(Z \leq -1) = 0.6217 - 0.1587 = 0.463$.
   (d) The 97.5th percentile of $Z$ is 1.96.

2. Let $Y$ = student heart rate. Then $Z = (Y - 95)/18.5$ follows a standard normal distribution.
   (a) $P(Y > 110) = P(Z > 0.81) = 1 - P(Z \leq 0.81) = 1 - 0.791 = 0.209$.
   (b) $P(90 < Y < 120) = P(-0.27 < Z < 1.35) = P(Z < 1.35) - P(Z \leq -0.27) = 0.5179$.
   (c) The first quartile (25th percentile) for a standard normal is $-0.67$. The corresponding heart rate is: $Y_{25} = 18.5 \times Z_{25} + 95 = 82.605$.
   (d) The third quartile (75th percentile) for a standard normal is 0.67. The corresponding heart rate is: $Y_{75} = 18.5 \times Z_{75} + 95 = 107.395$.
   (e) The 8th percentile for a standard normal is $-1.41$. The corresponding heart rate is: $Y_8 = 18.5 \times Z_8 + 95 = 68.915$.
   (f)

$$\begin{aligned} P(Y < 125 | Y > 100) &= \frac{P(100 < Y < 125)}{P(Y > 100)} \\ &= \frac{P(0.27 < Z < 1.62)}{P(Z > 0.27)} \\ &= \frac{P(Z < 1.62) - P(Z \leq 0.27)}{1 - P(Z \leq 0.27)} \\ &= \frac{0.9474 - 0.6064}{1 - 0.6064} \\ &= 0.8664. \end{aligned}$$

3. Let $Y$ = Stanford-Binet IQ score. Then $Z = (Y - 100)/15$ follows a standard normal distribution.
   (a) The 98th percentile for a standard normal is 2.05. The corresponding IQ score is: $Y_{98} = 15 \times Z_{98} + 100 = 130.75$. Thus the minimum IQ score needed is 130.75.
   (b) The 99th percentile for a standard normal is 2.33. The corresponding IQ score is: $Y_{99} = 15 \times Z_{99} + 100 = 134.95$.
   (c) $P(85 < Y < 115) = P(-1 < Z < 1) = P(Z < 1) - P(Z \leq -1) = 0.8413 - 0.1587 = 0.6826$.
   (d) Multiplication rule of two independent events: $0.02 \times 0.02 = 0.0004$.

4. (a) $\mu_Z = -2 + 4\mu_X = -2 + 4 \times 23 = 90$.
   (b) $\mu_Z = \mu_X + 2\mu_Y = 23 + 2 \times 6 = 35$.
   (c) $\sigma_Z^2 = (-2)^2\sigma_X^2 = 4 \times \frac{1}{4} = 1$.
   (d) $\sigma_Z^2 = \sigma_X^2 + (-2)^2\sigma_Y^2 = 2 + 4 \times \frac{1}{4} = 3$.

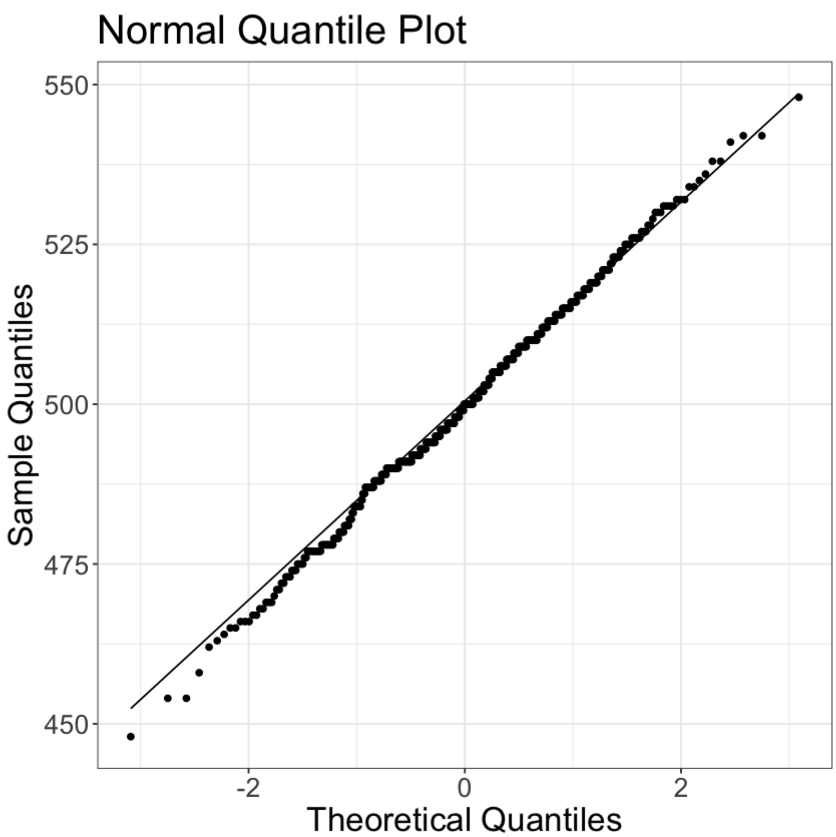
5. Applying the 68%-95%-99.7%, we expect approximately 99.7% of the daily rainfall to fall within the range of $-0.29$ inches to $0.31$ inches. However, since rainfall cannot be negative, there is a limitation on the possible values, and the distribution will be skewed towards higher values. Given this constraint, it is expected that the rainfall data will not exhibit the symmetric bell-shaped pattern typically seen in a normal distribution. The skewness in the data, due to the limitation on negative values, indicates that the daily rainfall at Sunshine City does not follow a normal distribution.

In [7]:
```
library(ggplot2)

# (a)
round(pbinom(489, size = 1000, prob = 0.5), 4)
round(1-pbinom(510, size = 1000, prob = 0.5), 4)

# (b)
y <- rbinom(500, size = 1000, prob = 0.5)
quantile(y, probs = c(0.25, 0.75))

# (c)
ggplot(data.frame(y = y),
       aes(sample = y)) +
  stat_qq() +
  stat_qq_line() +
  labs(x = "Theoretical Quantiles", y = "Sample Quantiles") +
  ggtitle("Normal Quantile Plot") +
  theme_bw() +
  theme(text = element_text(size = 20))
```

0.2533

0.2533

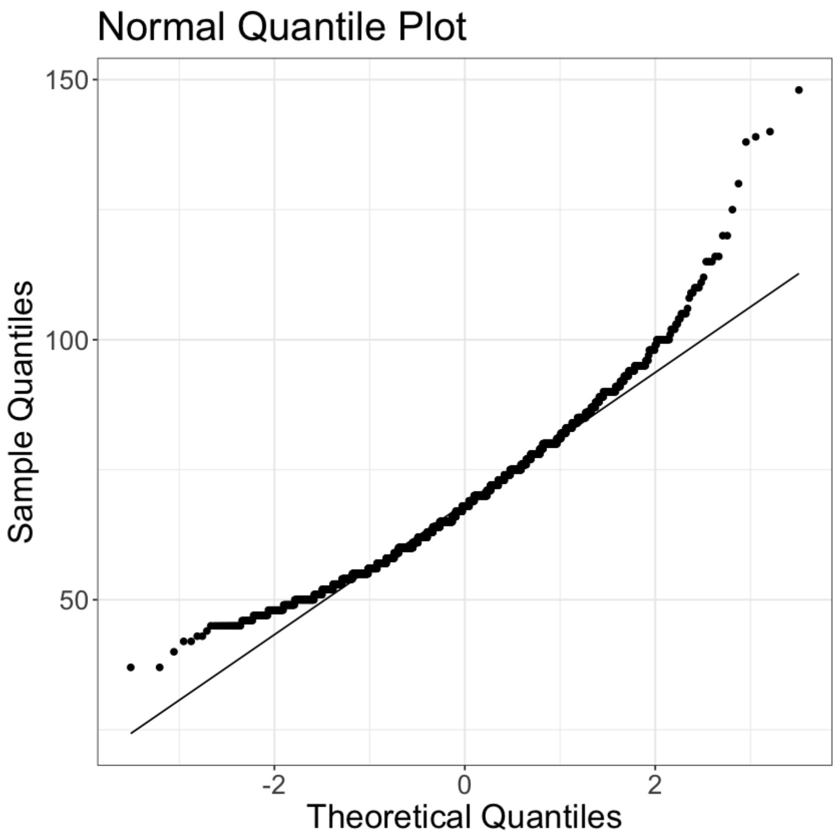
**25%:** 490 **75%:** 511




(c) The normal quantile plot appears fairly linear, suggesting that the observed values generally agree with the theoretical values and the normal model provides a reasonable approximation to the sample.

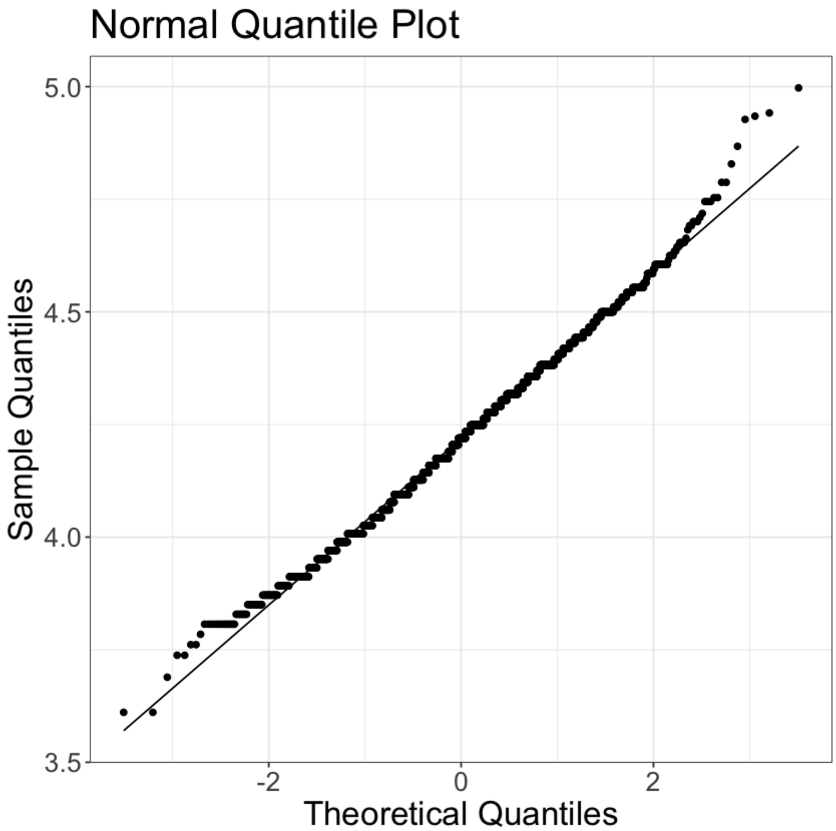
In [8]:
```
library(isdals); data(vitamina)
# (a)
```

```r
data(vitamina)
ggplot(vitamina, aes(sample = wt)) +
  stat_qq() +
  stat_qq_line() +
  labs(x = "Theoretical Quantiles", y = "Sample Quantiles") +
  ggtitle("Normal Quantile Plot") +
  theme_bw() +
  theme(text = element_text(size = 20))
```




(a) The normal quantile plot shows strong nonlinear patterns, indicating that the data do not agree with the normal distribution. The data is skewed to the right.

In [9]:
```r
# (b)
ggplot(vitamina, aes(sample = log(wt))) +
  stat_qq() +
  stat_qq_line() +
  labs(x = "Theoretical Quantiles", y = "Sample Quantiles") +
  ggtitle("Normal Quantile Plot") +
  theme_bw() +
  theme(text = element_text(size = 20))
```




(b) After the logarithmic transformation, the normal quantile plot appears fairly linear, indicating the data is better conformed with the normal distribution in the sense that. The logarithmic transformation is thus shown to be able to pull in the long right-hand tail and push out the short left-hand tail, making the distribution more nearly symmetric.